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# SECOND-ORDER BLIND SOURCE SEPARATION: A NEW EXPRESSION OF INSTANTANEOUS SEPARATING MATRIX FOR MIXTURES OF DELAYED SOURCES

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## ABSTRACT

In this paper, we study the blind separation of mixtures of propagating waves (delayed sources) encountered for example in underwater telephone (UWT) systems. We suggest a new second-order statistics method using as many observations as sources. First, we show that each of the  $N$  delayed sources can be developed as a particular linear combination of the different temporal-derivatives of the  $N$  observations. Under some assumptions, an instantaneous rectangular separating matrix is then identified by the joint diagonalization of a set of covariance matrices estimated from the observations and its derivatives. The algorithm used takes into account the particular structure of the spectral mixing matrix encountered. A numerical simulation is provided in a 3-sources/3- observations case for propagating audio signals.

## 1. INTRODUCTION

Consider a set of  $N$  propagating waves  $s_j(t)$  in an echo-free and noise-free medium. These are recorded on a set of  $M$  identical sensors. Let  $x_j(t)$  denote the contribution of the source  $s_j(t)$  on a sensor arbitrary indexed by 1, *i.e.*  $x_j(t)$  is a filtered version of the source  $s_j(t)$ . Assuming that the set of sensors is sufficiently compact, then the contribution of the source  $s_j(t)$  recorded on a sensor  $i$  is the same one as that recorded on the first sensor, except for an attenuation factor and a propagation delay. The observation  $y_i(t)$  recorded on the sensor  $i$ , is a linear combination of the delayed contribu-

tions  $x_j(t)$ :

$$\begin{aligned} y_1(t) &= \sum_{j=1}^N x_j(t), \\ y_i(t) &= \sum_{j=1}^N c_{ij} x_j(t - \tau_{ij}), \quad i \in [2, \dots, M], \end{aligned} \quad (1)$$

where  $c_{ij}$  is the relative attenuation coefficient of source  $j$  on sensor  $i$ , and  $\tau_{ij}$  a relative propagation delay of source  $j$  on sensor  $i$ .

The different contributions are assumed to be band-limited, differently colored, statistically independent and zero-mean. Thereafter in the paper, the contributions will be called sources.

The problem is to provide an estimation of the  $N$  different sources  $x_j(t)$  from the observations  $y_i(t)$ .

A first way to solve the problem using truncated Taylor series of each delayed source  $x_j(t - \tau_{ij})$  has been successfully treated in [1]-[5]. It consists of the estimate of an instantaneous square mixing matrix  $\mathbf{M}_{M \times M}$  between the observations and the different derivatives of the sources:

$$[y_1, y_2, \dots]^T = \mathbf{M} [x_1, x_2, \dots, \dot{x}_1, \dot{x}_2, \dots]^T.$$

For low delays, second-order statistics are sufficient to extract the different contributions from the observations (see [2], [4], [6]). For higher delays, a combination of two-order and high-order statistics methods is proposed in [3] to achieve the separation.

The main limitation of these methods is the high necessary number of observations (up to several times the number of sources).

We propose here a  $N$ -sources  $N$ -sensors original approach based on the direct estimation of an instantaneous rectangular

separating matrix  $\mathbf{S}$  between the different derivatives of the observations and filtered sources:

$$[x_1^h, x_2^h, \dots, x_N^h]^T = \mathbf{S} [y_1, \dot{y}_1, \dots, y_2, \dot{y}_2, \dots, y_N, \dot{y}_N, \dots]^T,$$

where  $x_j^h$  is the source  $x_j$  filtered by an invertible known filter  $h$ .

It is important to note that the two approaches are not equivalent: the matrix  $\mathbf{M}$  is not a simple pseudoinverse of  $\mathbf{S}$ .

The paper is organized as follows: in the next section we explain how to construct the separating matrix  $\mathbf{S}$  from the propagation model described by the system of equations (1). Then the third section explains the second-order statistics method implemented to estimate this separating matrix. The estimation uses a new version of the joint-diagonalization algorithm proposed by S.Dégerine in [7]. A description of this algorithm has to appear in english, a preprint can already be read in [8]. A numerical simulation in the last section illustrates the effectiveness of our approach.

## 2. EXPRESSION OF THE SEPARATING MATRIX

In the frequency domain, the system (1) becomes

$$\begin{aligned} Y_1(\nu) &= \sum_{j=1}^N X_j(\nu), \\ Y_i(\nu) &= \sum_{j=1}^N c_{ij} e^{-j2\pi\nu\tau_{ij}} X_j(\nu), \quad i \in [2, \dots, N]. \end{aligned} \quad (2)$$

$X_j(\nu)$  and  $Y_i(\nu)$  are respectively the Fourier Transforms (FT) of the  $j^{\text{th}}$  source and of the  $i^{\text{th}}$  observation.

The system (2) can be rewritten in matrix notation as:

$$\mathbf{Y}(\nu) = \mathbf{M}^f(\nu) \mathbf{X}(\nu), \quad (3)$$

where

$$\mathbf{Y}(\nu) = [Y_1(\nu), \dots, Y_N(\nu)]^T, \mathbf{X}(\nu) = [X_1(\nu), \dots, X_N(\nu)]^T.$$

The  $N \times N$  spectral mixing matrix  $\mathbf{M}^f(\nu)$  is defined as:

$$\mathbf{M}^f(\nu) = \begin{bmatrix} 1 & \dots & 1 \\ c_{21}e^{-j2\pi\nu\tau_{21}} & \dots & c_{2N}e^{-j2\pi\nu\tau_{2N}} \\ \vdots & \ddots & \vdots \\ c_{N1}e^{-j2\pi\nu\tau_{N1}} & \dots & c_{NN}e^{-j2\pi\nu\tau_{NN}} \end{bmatrix}. \quad (4)$$

We assume that  $\mathbf{M}^f(\nu)$  is regular for any frequency. The inverse matrix of  $\mathbf{M}^f(\nu)$  is

$$\mathbf{M}^f(\nu)^{-1} = \frac{1}{\det \mathbf{M}^f(\nu)} (\text{adj } \mathbf{M}^f(\nu))^T,$$

where  $\text{adj } \mathbf{M}^f(\nu)$  and  $\det \mathbf{M}^f(\nu)$  are respectively the adjoint matrix and the determinant of the matrix  $\mathbf{M}^f(\nu)$ .

From equation (3), one has:

$$\det \mathbf{M}^f(\nu) \mathbf{X}(\nu) = (\text{adj } \mathbf{M}^f(\nu))^T \mathbf{Y}(\nu). \quad (5)$$

According to the particular structure of the spectral mixing matrix  $\mathbf{M}^f(\nu)$ , its determinant (denoted by  $H_d(\nu)$ ) is a sum of weighted complex exponentials:

$$H_d(\nu) = \det \mathbf{M}^f(\nu) = \sum_i \beta_i \exp(-j2\pi\nu\tau_{d_i}),$$

where the weights  $\beta_i$  are particular products of coefficients  $c_{ij}$ , and  $\tau_{d_i}$  are particular sums of delays  $\tau_{ij}$ .

Let  $\mathbf{H}(\nu)$  denote the transposed adjoint of  $\mathbf{M}(\nu)$ . Each entries  $H_{kl}(\nu)$  of  $\mathbf{H}(\nu)$  can also be expressed as a sum of weighted exponentials:

$$H_{kl}(\nu) = (\text{adj } \mathbf{M}^f(\nu))_{kl}^T = \sum_i \alpha_{kli} \exp(-j2\pi\nu\tau_{kli}),$$

where the weights  $\alpha_{kli}$  are also particular products of coefficients  $c_{ij}$ , and  $\tau_{kli}$  are particular sums of delays  $\tau_{ij}$ .

Back to the temporal representation, the system (5) becomes:

$$\{h_d * x_k\}(t) = \sum_{l=1}^N \{h_{kl} * y_l\}(t),$$

where  $h_{kl}(t) = \text{FT}^{-1}(H_{kl}(\nu))$ ,  $h_d(t) = \text{FT}^{-1}(H_d(\nu))$  and where  $\{h * x\}(t) = \int h(t - \tau)x(\tau)d\tau$  denotes the convolution of  $h(t)$  and  $x(t)$ .

Each filtered source  $\{h_d * x_k\}(t)$  is then a finite sum of delayed observations:

$$\{h_d * x_k\}(t) = \sum_{l,i} \alpha_{kli} y_l(t - \tau_{kli}). \quad (6)$$

Assuming that the  $\tau_{kli}$  are “small” for all indices,  $y_l(t - \tau_{kli})$  can be approximated by its truncated up to P-order Taylor series expansion. Each filtered source can be then approximated by a linear combination of the observations and its derivatives:

$$\{h_d * x_k\}(t) \approx \sum_{l,i} \alpha_{kli} \sum_{p=0}^P \frac{(-\tau_{kli})^p}{p!} y_l^{(p)}(t).$$

For  $k = 1, \dots, N$  the system is rewritten as:

$$\mathbf{x}^h(t) \approx \mathbf{S} \tilde{\mathbf{y}}(t), \quad (7)$$

where  $\mathbf{x}^h(t) = [x_1^h(t), \dots, x_N^h(t)]^T$  is the filtered sources vector, with  $x_k^h(t) = \{h_d * x_k\}(t)$ ,

$\tilde{\mathbf{y}}(t) = [y_1, \dot{y}_1, \dots, y_1^{(P)}, y_2, \dots, y_2^{(P)}, \dots, y_N^{(P)}]^T$  is the observations and their derivatives vector.  $\mathbf{S}$  is the  $N \times (NP + N)$  separating matrix.

The analytical expression of each entry of  $\mathbf{S}$  with respect to the parameters  $c_{ij}$  and  $\tau_{ij}$  is heavy but does not present theoretical difficulties. This expression is detailed in the Appendix A for the 2-sources/2-sensors case.

The joint estimation of the parameters  $c_{ij}, \tau_{ij}$  and of the separating matrix  $\mathbf{S}$  forms the basis of the iterative algorithm proposed in the following section.

### 3. PRESENTATION OF THE ALGORITHM

From (7) and because the sources  $x_k^h(t)$  are mutually uncorrelated, it follows that  $\mathbf{S}$  has to diagonalize the set of covariance matrices obtained at different lags  $\mathbf{R}_{\tilde{\mathbf{y}}\tilde{\mathbf{y}}}(\tau_k) = \mathbb{E}\{\tilde{\mathbf{y}}(t)\tilde{\mathbf{y}}^T(t + \tau_k)\}$ :

$$\mathbf{S}\mathbf{R}_{\tilde{\mathbf{y}}\tilde{\mathbf{y}}}(\tau_k)\mathbf{S}^T = \mathbf{\Lambda}(\tau_k), \quad \forall \tau_k$$

$\mathbf{\Lambda}(\tau_k) = \mathbb{E}\{\mathbf{x}^h(t)\mathbf{x}^{hT}(t + \tau_k)\}$  being a diagonal matrix whatever the lag  $\tau_k$ .

S. Dégerine propose in [7],[8] a new algorithm (called Least Square on B or LSB) for approximate non-orthogonal joint diagonalization of a set of matrices. In its original form, this algorithm iteratively searches an optimal joint diagonalizer  $\tilde{\mathbf{S}}$  of a set of  $K$  matrices to the mean square sense. The second-order criterion to be optimized is then

$$C(\tilde{\mathbf{S}}) = \sum_k \text{Off}(\tilde{\mathbf{S}}\mathbf{R}_{\tilde{\mathbf{y}}\tilde{\mathbf{y}}}(\tau_k)\tilde{\mathbf{S}}^T), \quad (8)$$

where  $\text{Off}(\cdot)$  is the sum of the square non diagonal elements of the considered matrix. To perform the mean square optimization, S. Dégerine uses a relaxation on the lines of  $\tilde{\mathbf{S}}$  conducting to solve  $N$  eigenvalue problems for each iteration.

Here, the expected diagonalizer  $\mathbf{S}$  being non square, the optimization (8) does not conduct to a unique solution for  $\tilde{\mathbf{S}}$ . The main idea is then to force the algorithm to hold the theoretical structure of the separation matrix at each iteration. This theoretical expression depends on the parameters  $c_{ij}$  and  $\tau_{ij}$ . At each step of relaxation, we estimate the parameters  $c_{ij}$  and  $\tau_{ij}$  from their formal expression wrt the entries of the current estimated  $\tilde{\mathbf{S}}$ . The next step is processed using a new  $\tilde{\mathbf{S}}$  built on these estimated parameters.

An illustration of such theoretical expressions can be found in Appendices for the 2-sources case. For the  $N$ -sources case, we implemented formal calculus subroutine in order to provide automatically the formal expressions we need.

The final algorithm is summed up as follows:

#### Numerical Algorithm

- 1) From the observations  $\mathbf{y}(t)$ , build the observation vector  $\tilde{\mathbf{y}}(t)$  at the order  $P$ .
- 2) Compute  $K$  covariance matrix  $\mathbf{R}_{\tilde{\mathbf{y}}\tilde{\mathbf{y}}}(\tau_k)$ .
- 3) Initialize  $\tilde{\mathbf{S}}(0)$  with arbitrary values.
- 4) Proceed to an iteration  $k$  with LSB algorithm.
- 5) From  $\tilde{\mathbf{S}}(k)$  obtained at step 4), estimate the attenuation coefficients  $c_{ij}^k$  and the propagation delays  $\tau_{ij}^k$ .
- 6) From  $c_{ij}^k$  and  $\tau_{ij}^k$ , compute a new matrix  $\tilde{\mathbf{S}}(k + 1)$ .
- 7) Repeat steps 4), 5) and 6) until the evolution of the estimated matrix  $\tilde{\mathbf{S}}$  becomes sufficiently weak.

### 4. RESULTS

We present now a numerical simulation in the 3-sources/3-observations case. The sources are 24s long music extracts. The sampling frequency is  $F_s = 22.05 \text{ kHz}$ .

The matrix  $\mathbf{C}$  of relative attenuations is:

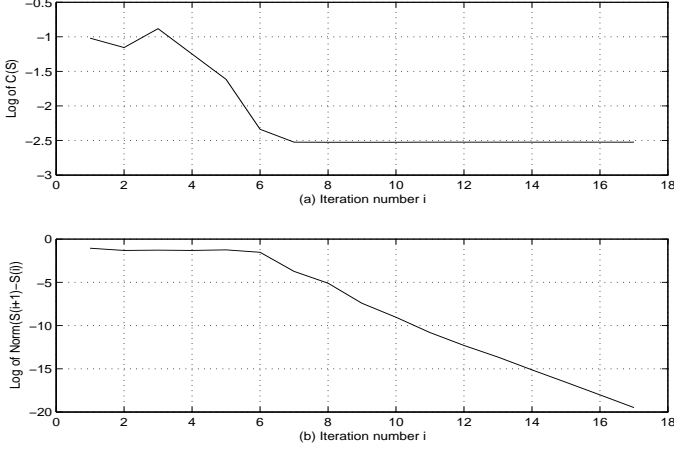
$$\mathbf{C} = \begin{bmatrix} 1.0000 & 1.0000 & 1.0000 \\ 2.0000 & 1.3000 & 1.2000 \\ 3.0000 & 1.6000 & 2.2000 \end{bmatrix}.$$

The matrix  $\mathbf{D}$  of relative propagation delays is:

$$\mathbf{D} = \begin{bmatrix} 0 & 0 & 0 \\ 30.00\mu\text{s} & 18.00\mu\text{s} & 5.00\mu\text{s} \\ 25.00\mu\text{s} & 12.50\mu\text{s} & 10.00\mu\text{s} \end{bmatrix}.$$

For the 3 observations, we take a Taylor series expansion up to 3-order ( $P = 3$ ). 151 covariance matrices  $\mathbf{R}_{\tilde{\mathbf{y}}\tilde{\mathbf{y}}}(\tau_k)$  are used ( $K = 151$ ), with  $\tau_k = k/F_s$ ,  $k \in \{-75, 75\}$ .

Fig. 1 (a) shows the evolution of the optimization criterion  $C(\tilde{\mathbf{S}})$  at each iteration number  $i$  and part (b) presents the Frobenius norm of the matrix  $\tilde{\mathbf{S}}(k + 1) - \tilde{\mathbf{S}}(k)$



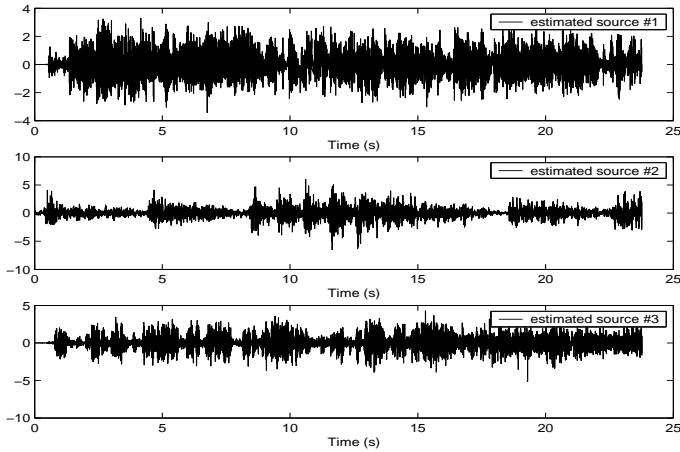
**Fig. 1.** Convergence

After convergence (8 iteration steps), we obtain the following estimations for the relative attenuations and for the relative propagation delays:

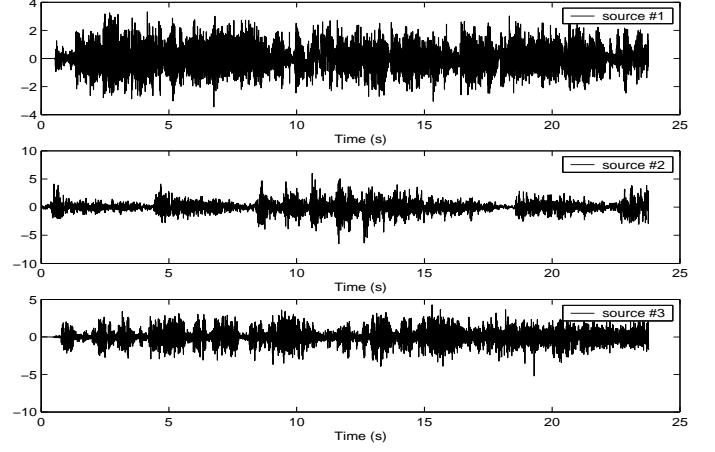
$$\tilde{\mathbf{C}} = \begin{bmatrix} 1.0000 & 1.0000 & 1.0000 \\ 1.9995 & 1.3047 & 1.1967 \\ 2.9998 & 1.6053 & 2.1929 \end{bmatrix},$$

$$\tilde{\mathbf{D}} = \begin{bmatrix} 0 & 0 & 0 \\ 31.64\mu s & 18.87\mu s & 5.03\mu s \\ 26.56\mu s & 13.13\mu s & 9.81\mu s \end{bmatrix}.$$

Taking the matrices  $\tilde{\mathbf{C}}$  and  $\tilde{\mathbf{D}}$  into account, the filter  $H_d(\nu)$  can be identified and its effect reversed after separation. We obtain the following source estimations (Fig. 2) that have to be compared to the original ones (Fig. 3).



**Fig. 2.** Estimated sources



**Fig. 3.** Original sources

The mean square errors (MSE) between the estimated sources and the original ones and the corresponding Signal to Interference Ratio (SIR) are presented in the Table 1.

	source #1	source #2	source #3
MSE	0.0141	0.0155	0.0164
SIR	41 dB	44 dB	36 dB

**Table 1.** Performances

## 5. CONCLUSION

This paper present a novel iterative method for blind source separation from delayed mixtures. The delayed mixtures are approached by instantaneous mixtures of the observation derivatives. This approach uses only second-order statistics and needs no more mixtures than sources. The algorithm is based on the joint diagonalization of a set of spatial covariance matrices but the non square separating matrix is constrained to hold a theoretical structure at each iteration. A numerical simulation shows the efficiency of the approach.

## APPENDIX A

FROM  $\mathbf{M}^f(\nu)$  TO  $\mathbf{S}$  IN THE 2-OBSERVATIONS CASE.

Now, we have the following spectral mixing matrix

$$\mathbf{M}^f(\nu) = \begin{bmatrix} 1 & 1 \\ c_{21}e^{-j2\pi\nu\tau_{21}} & c_{22}e^{-j2\pi\nu\tau_{22}} \end{bmatrix}. \quad (9)$$

In this case (6) leads to the 2 following filtered sources:

$$\begin{aligned} x_1^h(t) &= c_{22}y_1(t - \tau_{22}) - y_2(t), \\ x_2^h(t) &= -c_{21}y_1(t - \tau_{21}) + y_2(t), \end{aligned}$$

where  $x_k^h(t) = x_k(t - \tau_{22}) - x_k(t - \tau_{21})$ ,  $k \in [1, 2]$ .

With  $\mathbf{x}^h(t) = [x_1^h, x_2^h]^T$ , and  $\tilde{\mathbf{y}}(t) = [y_1, \dot{y}_1, \ddot{y}_1, \dots, y_1^{(P)}, y_2]^T$ , the  $(2 \times P + 2)$  unmixing matrix of equation (7) becomes:

$$\mathbf{S} = \begin{bmatrix} c_{22} & -c_{22}\tau_{22} & \dots & c_{22}\frac{(-\tau_{22})^P}{P!} & -1 \\ -c_{21} & c_{22}\tau_{21} & \dots & -c_{21}\frac{(-\tau_{21})^P}{P!} & 1 \end{bmatrix}.$$

## APPENDIX B

FROM  $\mathbf{S}$  TO  $c_{ij}, \tau_{ij}$  IN THE 2-OBSERVATIONS CASE.

Here, we have the following separating matrix

$$\mathbf{S} = \begin{bmatrix} s_{11} & s_{12} & \dots & s_{1P} & s_{1P+1} \\ s_{21} & s_{22} & \dots & s_{2P} & s_{2P+1} \end{bmatrix},$$

one has the  $(2 \times 2)$  spectral separating matrix

$$\mathbf{S}^f(\nu) = \begin{bmatrix} s_{11}^f(\nu) & s_{12}^f(\nu) \\ s_{21}^f(\nu) & s_{22}^f(\nu) \end{bmatrix},$$

where the entries of  $\mathbf{S}^f(\nu)$  are polynomials in  $\nu$ :

$$\begin{aligned} s_{11}^f(\nu) &= \sum_{k=1}^P (2j\pi\nu)^k s_{1k}, \\ s_{12}^f(\nu) &= s_{1P+1}, \\ s_{21}^f(\nu) &= \sum_{k=1}^P (2j\pi\nu)^k s_{2k}, \\ s_{22}^f(\nu) &= s_{2P+1}. \end{aligned}$$

The inverse matrix of  $\mathbf{S}^f(\nu)$  is also a matrix of polynomials:

$$[\mathbf{S}^f(\nu)]^{-1} = \frac{1}{\det \mathbf{S}^f(\nu)} \begin{bmatrix} p_{11}(\nu) & p_{12}(\nu) \\ p_{21}(\nu) & p_{22}(\nu) \end{bmatrix}.$$

An approximation of the spectral mixing matrix  $\mathbf{M}^f(\nu)$  is obtained normalizing each column of the previous matrix by  $p_{1k}(\nu)$

$$\mathbf{M}^f(\nu) \approx \begin{bmatrix} 1 & 1 \\ p_{21}(\nu)/p_{11}(\nu) & p_{22}(\nu)/p_{12}(\nu) \end{bmatrix}.$$

The entries of the second line of this matrix can be developed in power series expansion in  $\nu$ :

$$\begin{aligned} p_{21}(\nu)/p_{11}(\nu) &= q_0 + q_1\nu + O^2(\nu), \\ p_{22}(\nu)/p_{12}(\nu) &= r_0 + r_1\nu + O^2(\nu). \end{aligned}$$

Identifying these expansions with the Taylor-series expansion of the entries of  $\mathbf{M}^f(\nu)$  (see 9) we find:

$$\begin{aligned} c_{21} &= q_0, \\ \tau_{21} &= -\frac{1}{j2\pi} \frac{q_1}{q_0}, \\ c_{22} &= r_0, \\ \tau_{22} &= -\frac{1}{j2\pi} \frac{r_1}{r_0}. \end{aligned}$$

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