

# TMA from Cosines of Conical Angles Acquired by a Towed Array

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## Article TMA from cosines of conical angles acquired by a towed array

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Abstract: This paper deals with the estimation of the trajectory of a target in constant velocity mo-12 tion at an unknown constant depth, from measurements of conical angles supplied by a linear array. 13 Sound emitted by the target does not navigate necessarily along a direct path toward the antenna, 14but can bounce on the sea bottom and/or on the surface. Observability is thoroughly analyzed to 15 identify the ghost targets before proposing an efficient way to estimate the trajectory of the target 16 of interest and of the ghost targets when they exist. 17

Keywords: Target motion analysis, observability, Fisher information matrix, Cramér-Rao lower 18 bound, conical angles, nonlinear estimation. 19

## 1. Introduction

Bearings-only target motion analysis (BOTMA) is a problem that has been widely 22 studied and various solutions have been proposed in the literature: batch [1-5] or recursive 23 filter (such as extended Kalman filter [6-8], unscented Kalman filter [9], particle filter [10], 24 modified instrumental variable [11-13]), or a mix of recursive and batch methods [14]. 25 Citing all the papers dealing with this topic is now a hard task. Among the abundant lit-26 erature, most papers share the same assumption: the target is moving in a straight line 27 with a constant speed, while the passive observer is maneuvering adequately in order to 28 ensure observability of the target [15-17]. The bearings are the measurements. 29

In this paper we are concerned with the same problem except that the available meas-30 urements are the cosine of the relative bearings, called also conical angles because the 31 target belongs to the cone of ambiguity whose revolution axis is the line along which the 32 towed array is moving (see [18] p. 39). Implicitly, we consider a target moving in 3D at a 33 constant and unknown depth in near field; in this case, the two more energetic rays are 34 the direct and the reflected paths (bottom or surface). Most of the cases, the sound is 35 bounced by the sea bottom. Therefore, we extend our analysis to surface and sea bottom 36 bounced rays. 37

Indeed, the array detects the cosine of the relative angle of the direction of arrival by 38 a suitable spatial filtering such as beamforming, or more sophisticated techniques (see 39 [19]). In the near field, sound can propagate to the sensor array along the direct path 40 and/or the bottom-reflected path, and/or the surface-reflected path. Most of the time, at 41 most two rays coming from the same target are detected [18, 20]. 42

Unlike Gong [21] and Blanc-Benon [22], who address the three-dimensional target 43 motion analysis (TMA) from a sequence of time differences of arrival (TDOA) of a signal 44

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traveling by two different paths coupled with a sequence of azimuths, we assume in this 45 paper that the available measurements are the cosines of the conical angles only. In [23], 46 a similar problem was addressed, but observability was not studied. We will consider two 47 situations: the first case is devoted to TMA when sound propagates along a non-direct 48 path at each sampling time. This will be the topic of section 3: we will conduct observabil-49 ity analysis and identify all the ghost targets, given a set of noise-free measurements. We 50 will prove that an assumption on the target's depth makes the target's trajectory observa-51 ble, but not estimable (in the sense that the asymptotic performance given by the Cramér-52 Rao lower bound – CRLB - of the estimator of the depth is out of the physical constraints, 53 that is, the source is navigating between the surface and the sea bottom). 54

In the fourth section, we will consider scenarios in which the antenna changes its own route. We will prove that the trajectory of the target is almost certainly observable.

In the fifth section, we will assume that sound will propagate along the direct path 57 and the bottom-reflected path. The two rays will be assumed detected. Observability anal-58 ysis will reveal that only three ghost targets at most exist without maneuvering of the 59 antenna. We will check that in this case the depth is not "estimable". We will give a palli-60 ative allowing us to propose an estimator which is operationally acceptable, the price be-61 ing a small bias. Convincing simulations will be given at the end of this section, proving 62 that, even when the duration of the scenario is short, the estimated trajectory is very close 63 to the true one. A conclusion ends the paper. 64

## 2. Notation and problem formulation

We consider two underwater vehicles moving at their own constant depth. The first 66 mobile is a surface vessel or a submarine towing a horizontal sensor array, and the second 67 one is the target of interest. Given a Cartesian coordinate system, the acoustic center of the 68 array is located at time t at  $(x_0(t) y_0(t) z_0)^T$ . At the same time, the target is at 69  $(x_T(t) \quad y_T(t) \quad z_T)^T$ . The respective horizontal positions of the target of interest and of 70 the center of the array at time t are denoted by  $P_T(t) = (x_T(t) \ y_T(t))^T$ and 71  $P_0(t) = (x_0(t) \quad y_0(t))^T$ . The sea bottom depth (assumed to be a constant) is denoted D. 72 The source is said to be endfire to the line array if its trajectory is in the same line as the 73 array (which implies that the array and the source are at the same depth, and share the 74 same route). It is broadside to the antenna if it navigates in the vertical plane orthogonal 75 to the line array and passing by the acoustic center of the array. The sensors array detects 76 the line of sight of the target; more precisely, an ad-hoc array processing (or spatial filter-77 ing) delivers at time t the cosine of the conical angle  $c_a(t)$  given by 78  $\cos(c_a(t)) = \cos(\theta(t) - h(t))\cos(\phi(t)) \triangleq m(t)$ , where  $\theta(t)$  and  $\phi(t)$  are respectively 79 the azimuth (or bearing) and the elevation of the path along which the sound emitted by 80 the source propagates. The angle h(t) is the heading of the sensor array. Denoting the 81 relative position coordinates of the source with reference to the acoustic center of the array 82  $y_{OT}(t) = y_T(t) - y_O(t)$ by  $x_{OT}(t) = x_T(t) - x_O(t)$ and we have 83  $\theta(t) = \arctan(x_{0T}(t), y_{0T}(t))$ . Figure 1 displays the different angles and the two actors 84 (the observer reduced to the linear array, and the target). 85





The ray of the sound (or signal) emitted by the source can be reflected by the bottom 88 and/or the surface or travels in the surface or deep channel. The sound-speed profile 89 makes the paths curve. In this paper, we will consider that the target is in the near field 90 (the distance between the source and the array is less than 20 km), and the bottom depth 91 is in the range [2000 m, 5000 m]. Due to the large curvature of the ray (about 80 km), we 92 will approximate the path of the sound as a set of zigzags defined by the reflections on 93 the bottom or on the surface. So, we implicitly use the Snell law widely employed in geo-94 metrical optics. An image-source is created whose depth  $\zeta_T$  will be called "image-depth". 95 A path is then defined by the triplet  $(\delta, n_B, n_S)$ , where 96

- δ indicates the direction of the path of the sound emitted by the source: if the path is 97 toward the surface, δ = -1, otherwise δ = +1, 98
- $n_B$  is the number of bottom reflections, and 99
- *n<sub>s</sub>* is the number of surface reflections.

Figure 2 illustrates three different paths



**Figure 2**. Three examples of ray path: in solid line, the direct path  $(\delta, n_B, n_S) = (+1,0,0)$ , in dashed - 103 dotted line the bottom reflected path  $(\delta, n_B, n_S) = (+1,1,0)$ , and in dashed line the bottom-surface - 104 bottom reflected path  $(\delta, n_B, n_S) = (+1,2,1)$ . 105

We have to consider the depth difference between the array and the image-source 106 defined by  $\zeta_{0T} \triangleq \zeta_T - z_0$  if the ray has been reflected (by the sea bottom or by the surface), or  $\zeta_{0T} \triangleq z_T - z_0$  if the sound wave uses the direct path. 108

A general expression of  $\zeta_{0T}$  based on the triplet  $(\delta, n_B, n_S)$  is given by 109  $\zeta_{0T}(\delta, n_B, n_S) = -2\delta n_B(-1)^{n_S+n_B}D - z_0 + (-1)^{n_S+n_B}z_T$ . Note that, given the path, the link 110 between  $\zeta_{0T}(\delta, n_B, n_S)$  and  $z_T$  is linear:  $\zeta_{0T}(\delta, n_B, n_S) = az_T + b$ , the constants being a 111 function of the triplet  $(\delta, n_B, n_S)$ , D, and  $z_0$ . Moreover,  $\zeta_{0T}(\delta, n_B, n_S)$  is null if and only if 112

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the antenna and the target are navigating at the same depth ( $z_T = z_0$ ), and sound is traveling in the direct path. In this case,  $\cos(\phi(t)) = 1$ . For the sake of simplicity of the notations, we will simply subsequently denote  $\zeta_{0T}$  instead of  $\zeta_{0T}(\delta, n_B, n_S)$ .

For the above examples, we have  $\zeta_{OT}(1,0,0) = z_T - z_0$  (direct path), 116  $\zeta_{OT}(+1,1,0) = 2D - (z_T + z_0)$  (bottom-reflected path), and  $\zeta_{OT}(+1,2,1) = 4D - (z_T + z_0)$  117 (bottom-surface-bottom reflected path). Note that  $\zeta_{OT}(\delta, n_B, n_S)$  can be negative (the image-source is above the surface). Consequently, the cosine of the elevation is 119

$$\cos(\phi(t)) = \frac{\sqrt{x_{0T}^2(t) + y_{0T}^2(t)}}{\sqrt{x_{0T}^2(t) + y_{0T}^2(t) + \zeta_{0T}^2(\delta, n_B, n_S)}}.$$
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Figure 3 (a) displays the cone of ambiguity, defined by the set of sources sharing the 121 same  $\cos(\phi(t))$ . In Figure 3 (b), we have plotted a direct ray and a bottom-bounced ray, 122 which allows us to figure out the various angles with which we will work. 123



**Figure 3.** Cones of ambiguity (**a**) The cones that the target belongs to, and the one that the image-target belongs to. (b) Example of conical angles of the target and of the image-target, for a bottom-reflected ray:  $\phi^D$  and  $\phi^B$  are the elevations of the direct path and of the bottom-reflected path, respectively.

We assume that the source is moving in constant velocity (CV) motion during the scenario. Our challenge is to estimate its trajectory, i.e. the state vector defining it,  $X \triangleq (x_T(t^*) \ y_T(t^*) \ z_T \ \dot{x}_T \ \dot{y}_T)^T$ , for a chosen  $t^*$ , from noisy measurements. We consider two situations: 131

- 1. Only one ray is detected by the array during the scenario; in this case, we have at each time t a measurement m(t), given the path along which the wave propagates.
- 2. Two rays (traveling on two different paths) arrive at the sensors antenna. In this case, the available measurement at time *t* is a couple of measurements, say  $(m_1(t), m_2(t))$ , given the two paths along which the wave propagates.

After the spatial filtering, the antenna supplies a noisy measurement of m(t) or a 137 noisy measurement of  $(m_1(t), m_2(t))$ . The noisy measurements are regularly acquired at 138  $t_k = (k-1)\Delta t, \ k \in \{1, ..., N\}$ , for a fixed sampling time  $\Delta t$ . 139

Before attempting to estimate *X*, we must answer several questions:

1. Is the vector X observable from the set of measurements  $\{m(t), t \in [0, T]\}$ ? Note141that, in TMA problems, observability is often analyzed in continuous time (see [15]142and [17], for example), even though the noisy measurements are given in discrete143time.144

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2.	If not, what are the ghost targets (those which could be detected at the same set of	145
	measurements $\{m(t), t \in [0, T]\}$ ?	146

- 3. How do we make *X* observable or with which new information?
- 4. Is the vector X observable from the set of couples  $\{(m_1(t), m_2(t)), t \in [0, T]\}$ ?

For the cases where X is observable, we have then to compute the asymptotical performance of an unbiased estimator (given by the CRLB [24]), and the performance of our estimators in terms of bias and the covariance matrix. It is worth noting that using the FIM to prove observability can lead to a wrong conclusion [25]. This why we use an analytic approach. 149

## 3. TMA from one ray

In this section, we consider the case where the array collects the cosine of a conical 155 angle, the path of the ray being known by the operator. We start by analyzing the observability of the trajectory of the source of interest. 157

#### 3.1. Observability analysis

**Theorem 1.** Let a linear antenna measure the cosine of a conical angle in the direction of a source, both in CV motion. The path of the sound emitted by the source is known, as is also the sea bottom depth.

- 1. If the target is broadside to the antenna, then the set of ghost targets is composed of virtual 162 sources broadside to the antenna. 163
- If the target is endfire to the antenna, the set of ghost targets is composed of virtual sources 164 endfire to the antenna.
- 3. If the target has the same heading as the array (but is not endfire to it), then the set of ghost 166 targets is composed of virtual targets having the same heading as the antenna. More precisely, 167 the ghost image of each ghost target is moving on a cylinder whose axis is the antenna axis, 168 and whose radius is a positive scalar β. The relative ghost target velocity is equal to β times 169 the target's velocity. The initial distance between the ghost image and the center of the antenna 170 is equal to β times the initial distance between the target-image and the center of the antenna. 171
- 4. In any other cases, for a chosen image-depth  $\zeta_G$ , the set of ghost targets is composed of virtual 172 targets whose motion relative to the array is defined by  $P_{OG}(t) = \beta P_{OT}(t)$  or 173  $P_{OG}(t) = \beta S P_{OT}(t)$ , where **S** is the 2D axial symmetry around the line of the array, and  $\beta$  174 is a positive scalar. The scalar  $\beta$  is equal to  $\frac{|\zeta_{OG}|}{|\zeta_{OT}|}$  if  $\zeta_{OT} \neq 0$ . If  $\zeta_{OT} = 0$  (which can happen 175 with a direct path only),  $\beta$  can have any positive value. 176

Preamble: In the following proof, we choose  $t^* = 0$ . Instead of working with the state 177 vector  $X = (x_T(0) \ y_T(0) \ z_T \ \dot{x}_T \ \dot{y}_T)^T$ , we will use the relative state vector of the image source, which is  $Y \triangleq (x_{0T}(0) \ y_{0T}(0) \ \zeta_{0T} \ \dot{x}_{0T} \ \dot{y}_{0T})^T$ . The reason is that we are 179 able to recover X from Y without ambiguity. 180

We will prove this theorem in the special case where the heading of the antenna is equal to 0°, and the value  $y_{OT}(t)$  is positive. This can be easily obtained with an ad-hoc rotation of the whole scenario. This will simplify the expression of the measurement, without loss of generality.

**Proof of Theorem 1.** We are seeking the ghost target whose horizontal position at time *t* 185 is  $(x_G(t) \ y_G(t))^T$ , detected in the same cosine of the conical angle, that is 186

$$\frac{y_{OG}(t)}{\sqrt{x_{OT}^2(t) + y_{OT}^2(t) + \zeta_{OT}^2}} = \frac{y_{OG}(t)}{\sqrt{x_{OG}^2(t) + y_{OG}^2(t) + \zeta_{OG}^2}} , \quad \text{with} \quad x_{OG}(t) = x_G(t) - x_O(t) , \quad 187$$

 $y_{OG}(t) = y_G(t) - y_O(t)$ , and  $\zeta_{OG}$  is the image-depth of the ghost target. This equality is 188 equivalent to 189

$$\frac{y_{OT}^2(t)}{x_{OT}^2(t) + y_{OT}^2(t) + \zeta_{OT}^2} = \frac{y_{OG}^2(t)}{x_{OG}^2(t) + y_{OG}^2(t) + \zeta_{OG}^2}.$$
(1)

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Note that because the target is moving (as is the ghost target also), the denominators of 190 the left term and of the right term of (1) are two polynomial functions of degree 2. 191

<u>Case 1</u>:  $y_{0T}(t)$  is a zero function, i.e.  $\forall t \ y_{0T}(t) = 0$ . This means the source is broadside to the antenna:  $Y_T = (x_{0T}(0) \ 0 \ \zeta_{0T} \ \dot{x}_{0T} \ 0)^T$ . In this case,  $y_{0T}(t) = 0$ ,  $\forall t \in [0, T]$ . Hence, the set of ghost targets is composed of the virtual targets broadside to the antenna:  $Y_G = (x_{0G}(0) \ 0 \ \zeta_{0G} \ \dot{x}_{0G} \ 0)^T$ .

<u>Case 2</u>:  $y_{0T}(t)$  is not a zero function.

If  $\dot{y}_{0T} = 0$ , then  $y_{0T}(t)$  is a constant. To respect the degrees of the terms of (1),  $y_{0G}(t)$  is 197 a constant too. 198

If  $\dot{y}_{0T} \neq 0$ , then there is a root, say  $\tilde{t}$ , such as  $y_{0T}(\tilde{t}) = 0$ , since  $y_{0T}(t)$  is a polynomial 199 function of degree 1. Consequently,  $y_{OG}(\tilde{t}) = 0$ , and  $\forall t \neq \tilde{t}, y_{OG}(t) \neq 0$ . 200

We deduce that, in both cases ( $\dot{y}_{OT} = 0$ , and  $\dot{y}_{OT} \neq 0$ ), there exists a positive value  $\beta$  such 201 that  $y_{OG}(t) = \beta y_{OT}(t)$ . 202

$$(1) \Leftrightarrow \{ [x_{\partial G}^2(t) + y_{\partial G}^2(t) + \zeta_{\partial G}^2] - \beta^2 [x_{\partial T}^2(t) + y_{\partial T}^2(t) + \zeta_{\partial T}^2] \} y_{\partial T}^2(t) = 0 \Leftrightarrow x_{\partial G}^2(t) + \zeta_{\partial G}^2 = \beta^2 [x_{\partial T}^2(t) + \zeta_{\partial T}^2]$$

$$(2)$$

The quantity  $x_{0T}^2(t) + \zeta_{0T}^2$  can be equal to zero at any time, or at one time or never. Subcase 1:  $\forall t, x_{0T}^2(t) + \zeta_{0T}^2 = 0.$ 

Then  $x_{0T}(t) = 0$ ,  $\forall t$  and  $\zeta_{0T} = 0$ . Note that this case is the one when the target is travel-205 ing in the endfire to the array and at the same depth as the antenna and the path is the 206 direct one. For the same reason,  $x_{OG}(t) = 0$ ,  $\forall t$  and  $\zeta_{OG} = 0$ . The set of ghost targets is 207 hence composed of virtual targets traveling in the endfire to the array and at the same 208 depth as the antenna. 209

Subcase 2:  $\exists \check{t}$  such that  $x_{OT}^2(\check{t}) + \zeta_{OT}^2 \neq 0$ . 210

We deduce from (2) that

$$x_{OG}^{2}(0) = \beta^{2} x_{OT}^{2}(0) + \beta^{2} \zeta_{OT}^{2} - \zeta_{OG}^{2}$$
(3-a)

$$\begin{cases} x_{\bar{o}_{G}}(0) = \beta^{-} x_{\bar{o}_{T}}(0) + \beta^{-} \zeta_{\bar{o}_{T}}^{-} - \zeta_{\bar{o}_{G}}^{-} \\ x_{o_{G}}(0) \dot{x}_{o_{G}} = \beta^{2} x_{o_{T}}(0) \dot{x}_{o_{T}} \\ \dot{x}_{\bar{o}_{G}}^{2} = \beta^{2} \dot{x}_{\bar{o}_{T}}^{2} \end{cases}$$
(3-b)

(3-c)

If  $\dot{x}_{OT} = 0$ , then 212  $Y_G = \left(\pm \sqrt{\beta^2 x_{OT}^2(0) + \beta^2 \zeta_{OT}^2 - \zeta_{OG}^2} \quad \beta y_{OT}(0) \quad \underline{\zeta_{OG} \quad 0 \quad \beta \dot{y}_{OT}}\right)^T, \text{ for any positive constant}$ 213  $\beta$  and any positive constant  $\zeta_{0G}$  less than  $\sqrt{\beta^2 x_{0T}^2(0) + \beta^2 \zeta_{0T}^2}$ . Note that, when  $\dot{y}_{0T} = 0$ , 214 the target is motionless relative to the center of the array (both have the same velocity); 215 and when  $\dot{y}_{0T} \neq 0$ , the target has the same heading as the array. 216

If  $\dot{x}_{oT} \neq 0$ , then squaring the elements of (3-b), and using (3-c), we draw from (3-a) that 217  $\beta^2 \zeta_{OT}^2 = \zeta_{OG}^2$ . If  $\zeta_{OT} = 0$ , then  $\zeta_{OG} = 0$ , and the scalar  $\beta$  can take any positive value; else 218  $\beta = \frac{|\zeta_{OG}|}{|\zeta_{OT}|}$ . In both cases, the trajectory of a ghost target is defined by the state vector 219  $Y_G = (\pm \beta x_{OT}(0) \quad \beta y_{OT}(0) \quad \beta \zeta_{OT} \quad \pm \beta \dot{x}_{OT} \quad \beta \dot{y}_{OT})^T. \Box$ 220

Remarks:

- 1. When the source and the observer are at the same depth, and the path is direct, The-222 orem 1 recovers the conclusions given in [26]. 223
- 2. The cases (1), (2) and (3) of Theorem 1 are "rare events" since the events of dealing 224 with a source in endfire, broadside or having the same heading as the antenna during 225 the scenario occur with a probability equal to 0. However, when the target has a tra-226 jectory close to one of these special cases, the estimates will have a poor behavior. 227
- 3. For cases (4), when the detected ray is not a direct path, for example, when the ray is 228 bottom-reflected, a hypothesis about the source is sufficient to get one solution, cor-229 responding to a ghost target. Indeed, if we suppose that the depth of the target is  $z_{AS}$ 230 (whereas the true value is  $z_T$ ), then we have  $\beta = \frac{2D - (z_{AS} + z_0)}{2D - (z_T + z_0)}$ , whose biggest value 231

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 $\beta_{Max} = \frac{2D - z_0}{2D - (z_T + z_0)}$ , and the minimum value is  $\beta_{Min} = \frac{2D - (z_{Max} + z_0)}{2D - (z_T + z_0)}$  where  $z_{Max}$  is the 232 largest depth of a submarine vehicle. Typically, in deep water,  $D \ge 4000$ m. A rea-233 sonable choice of  $z_{Max}$  could be 400 m. We can then have a range of  $\beta$ :  $[\beta_{Min}, \beta_{Max}] = \left[\frac{7600-z_0}{8000-(z_T+z_0)}, \frac{8000-z_0}{8000-(z_T+z_0)}\right]$ . For instance, when the depths of the an-234 235 tenna and the target are respectively 200 m and 100 m, we have 236  $[\beta_{Min}, \beta_{Max}] = [0.974, 1.013]$ . In this way, we bound the set of ghost targets, and we 237 can expect that the bias induced by a wrong choice of  $z_{As}$  is very low. 238

For cases (4) again, with a direct path, if the target is not at the same depth as the 4. 239 antenna,  $\beta = \frac{z_{As} - z_0}{z_T - z_0}$ . Because  $\beta$  is a positive number,  $z_{As} - z_0$  has the same sign as 240  $z_T - z_0$ : if  $z_T > z_0$ , then  $z_0 < z_{AS} \le z_{Max}$ , and  $[\beta_{Min}, \beta_{Max}] = \left[0, \frac{z_{Max} - z_0}{z_T - z_0}\right]$ ; if  $z_T < z_0$ , 241 then  $0 \le z_{As} < z_0$ , and  $[\beta_{Min}, \beta_{Max}] = \left[0, \frac{z_0}{z_T - z_0}\right]$ . In both cases, the range  $[\beta_{Min}, \beta_{Max}]$ 242 is too wide to be useful. 243

If the target and the antenna are at the same depth,  $\beta$  can take any positive value.

#### 3.2. Estimation of the trajectory

We run 500 Monte Carlo simulations for a typical scenario described as follows: 246

The observer starts from  $(0 \ 0)^T$  at the depth  $z_0 = 200$ m. Its speed and heading are respectively 5 m/s and 0°. The initial position of the target is  $(5000 \quad 7000)^T$  and its depth is  $z_T = 100$ m. Its route is 45° and its speed is 4 m/s.

- The measurements are collected every 4 seconds ( $\Delta t = 4$ s). The scenario lasts 20 min. •
- The sea bottom depth is 4000 m. The detected ray is a bottom-reflected ray. •
- The assumed target depth is  $z_{As} = 200$  m (whereas the true one is 100 m). •
- First, the measurements have been corrupted with an additive Gaussian noise whose 253 standard deviation is  $\sigma = 1.7 \ 10^{-2}$ . 254

Then we choose the least squares estimator, which is identical to the maximum like-255 lihood estimator with these assumptions. Note that, in open literature about TMA, the 256 confidence regions are given by the confidence ellipsoid obtained with the covariance ma-257 trix of the estimate. Since the maximum likelihood estimate is asymptotically efficient un-258 der nonrestrictive conditions, we use here the Cramér-Rao lower bound to compute such 259 confidence regions.

The result of the simulation is presented in Table 1 and illustrated in Figure 4. Obvi-261 ously, even if the assumption made on the target's depth makes the state vector observa-262 ble, it remains inestimable: the hugeness of the diagonal elements of the CRLB does not 263 allow this kind of TMA to be employed. We note in Figure 4 that the cloud of horizontal 264 estimates is hyperbola-shaped. This is because the state vector is "weakly" estimable. The 265 parametric equation of this hyperbola is 266

$$\begin{cases} x(\omega) = \zeta_{AS} \operatorname{sinn}(\omega) \\ y(\omega) = \frac{\zeta_{AS}m}{\sqrt{1-m^2}} \operatorname{cosh}(\omega), \text{ with } \zeta_{AS} = 2D - (z_{AS} + z_0), \text{ and } m = \frac{y_{OT}(0)}{\sqrt{x_{OT}^2(0) + y_{OT}^2(0) + \zeta_{OT}^2}}.$$

We further reduced the standard deviation to  $\sigma = 1.7 \ 10^{-4}$  in order to appreciate 268 the behavior of the MLE. With this (unrealistic) value, the MLE is efficient, as shown in 269 Table 2 and in Figure 5 (which validates our observability analysis). 270

Our conclusion is that the state vector is not estimable, even though it is observable 271 with an assumption on the target's depth. 272

This is why we propose to maneuver the antenna in order to render the state vector 273 observable with no assumption on the target's depth, and to augment the information 274 about it. 275

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**Figure 4**. The cloud of estimated position (in green), a piece of the hyperbola (intersection of the cone of ambiguity and the plane  $z = z_{AS} (= 200 \text{ m})$ , for  $\sigma = 1.7 \text{ } 10^{-2}$ .

**Table 1.** Performance of the estimator of the reduced state vector when  $\sigma = 1.7 \ 10^{-2}$ , in279terms of bias, sample standard deviation and the one given by the square root of the diagonal of the CRLB280

$X_r$	Bias	$\sigma_{samp}$	$\sigma_{CRLB}$
5000 m	-3525	6962	13356
7000 m	-2367	4052	5599
2.83 m/s	-1.37	1.81	4.35
2.83 m/s	0.53	1.62	2.75



**Figure 5**. The cloud of estimated position (in green) for  $\sigma = 1.7 \ 10^{-4}$ . The cloud is no longer hyperbola-shaped. The small black segment is the 90%-confidence ellipsoid. 283

**Table 2.** Performance of the estimator of the reduced state vector with  $\sigma = 1.7 \ 10^{-4}$ .

$X_r$	Bias	$\sigma_{samp}$	$\sigma_{CRLB}$
5000 m	60.40	138.67	133.56
7000 m	88.20	58.42	55.99
2.83 m/s	0.043	0.044	0.044
2.83 m/s	0.037	0.028	0.028

## 4. TMA with one ray when the array maneuvers

In this section, the antenna maneuvers, i.e. it changes its own heading. We start by proving that the state vector is observable (without any assumption on the target's depth). 288

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Then we have recourse to Monte Carlo simulations to evaluate the performance of the 289 MLE. 290

4.1. Observability Analysis

**Theorem 2.** Suppose the antenna's trajectory is composed of two successive legs at constant ve-292 locity (however with the same speed). Let the target be in CV motion. The linear array acquires the conical angles of the wave emitted from the target, the path of the ray being known as well as the 294 sea bottom depth. If the target is broadside or endfire to the antenna during a leg, then there is at 295 most a ghost target. Otherwise, there is no ghost target.

Due to its length, the proof of this theorem is given in the Appendix.

### 4.2. Estimation

In this subsection, we present the result of 500 Monte Carlo simulations that are run 299 to illustrate the behavior of the proposed estimators. First, we give the scenario used here. 300

The center of the array and the initial position of the source are respectively at 301  $(200)^T$  and  $(5000 \ 7000 \ 100)^T$  at the very beginning of the scenario. The speed (0) 0 302 of the array is a constant along the scenario and is equal to 5 m/s. The trajectory of the 303 array is composed of two legs linked by an arc of a circle. The first leg lasts 1 min 40 s, 304 during which the array's heading is 135°. Then the array turns to the right with a turn rate 305 equal to 20°/min to adopt a new heading equal to 270°. The duration of the maneuver is 306 hence equal to 6 min 44 s. The second leg lasts 5 min, so the total duration of the scenario 307 is 13 min and 20 s. Meanwhile, the target is navigating with a heading equal to 45° and a 308 speed of 4 m/s. The bottom depth is D = 4000m. 309

The state vector have to estimate we is hence 310  $X = (5000 \ 7000 \ 100 \ 2.83 \ 2.83)^T$ . 311

The array is assumed to measure the cosines of the conical angles of the bottom-re-312 flected path given by 313  $y_{OT}(t_k)$ 

$$m(t_k) = \frac{y_{0T}(t_k)}{\sqrt{x_{0T}^2(t_k) + y_{0T}^2(t_k) + [2D - (z_T + z_0)]^2}} + \varepsilon_k.$$
314  
ts are acquired every  $\Delta t = 4$ s, with  $t_k = (k - 1)\Delta t$ .

Measurements are acquired every  $\Delta t = 4s$ , with  $t_k = (k - 1)\Delta t$ .

The noise vector  $\varepsilon_k$  is assumed to be Gaussian, 0-mean and its standard deviation equal to  $\sigma = 1.7 \ 10^{-2}$ . The vectors  $\varepsilon_k$  are also assumed to be temporally independent. 317 Again, we choose the least squares estimator. 318

#### 4.2.1. Estimation of X

The 500 obtained estimates of the initial horizontal position are plotted in Figure 6, 320 together with the trajectory of the target, the 90%-confidence ellipse and the trajectory of 321 the array. Again, the view is from the sky. 322

The performance of the estimator (bias and standard deviation of each component) 323 is presented in Table 3. 324

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Figure 6. The cloud of the 500 initial positions estimates and the 90%-confidence ellipse.

Table 3. Performance of the estimator of the plain state vector

X	Bias	$\sigma_{samp}$	$\sigma_{CRLB}$
5000 m	-44.77	854.72	868.12
7000 m	-68.16	1162.1	1173.60
100 m	7.14	558.55	545.99
2.83 m/s	0.092	1.67	1.65
2.83 m/s	0.194	2.72	2.68

A convenient way to evaluate the behavior of an estimator is to compute the so-called 329 normalized estimation error squared (NEES) [27], defined as  $N_l = (\hat{X}_l - X)^{T} F(\hat{X}_l - X)$ , 330 where *F* is the FIM, and  $\hat{X}_l$  is the estimate computed at the *l*-th simulation. If  $\hat{X}_l$  is 331 Gaussian-distributed with X as the mathematical expectation and the CRLB as the covar-332 iance matrix, then  $N_l$  is Chi-square distributed with d degrees of freedom ( $\chi_d^2$ ), where d 333 is the dimension of X (here 5). From the central limit theorem, the averaged NEES 334  $N_{S} \triangleq \frac{1}{N_{Sim}} \sum_{l=1}^{N_{Sim}} N_{l}$  is approximately Gaussian; its mathematical expectation is *d*, and its 335 standard deviation equal to  $\sqrt{\frac{2d}{N_{Sim}}}$ . 336

From our simulations, we obtain  $N_S = 5.34$ .

In conclusion, the estimator can be declared efficient. However, the minimum stand-338 ard deviation of the target's depth is not compatible with the physical constraints: with 339 the standard deviation given in Table 3, the target could be up above the sea surface! 340 Therefore, a palliative of this is to impose a depth on the target. Indeed, we saw in sub-341 section 3.1 that a supposed depth creates a small bias in estimation of the horizontal posi-342 tion of the target. 343

## 4.2.2. Estimation of *X* reduced when the depth of the target is fixed.

Now, the third component of *X* has not to be estimated. The new state vector is the 345 denoted  $X_r = (x_T(0) \quad y_T(0) \quad \dot{x}_T \quad \dot{y}_T)^T$ . We impose that  $z_{AS} = 200$  m (whereas the true 346 depth is still 100 m). Hence, we introduce a bias. 347

Figure 7 displays the position's estimates in the same manner as Figure 6. The bias is 348 not visible to the naked eye. However, Table 4 reveals this bias, which may be acceptable 349 in a real situation. Even though the averaged NEES (=7.31) is out of its 90% confidence 350 interval, its value remains acceptable. 351

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10 6 North (km) 2 0 -2 10 East (km)

Figure 7. The cloud of the 500 initial positions estimates with the reduced state vector and the 90%-confidence ellipse.

$X_r$	Bias	$\sigma_{samp}$	$\sigma_{CRLB}$
5000 m	65.40	655.61	606.41
7000 m	93.05	831.75	762.56
2.83 m/s	0.034	1.71	1.58
2.83 m/s	0.063	2.76	2.52

The main interest of assuming the depth to be known is to economize on the CPU 356 time, and reduce the standard deviation of the remaining components to estimate. We are 357 in the presence of the well-known bias-variance tradeoff. 358

4.2.3. Estimation of the reduced state vector by the conventional BOTMA

In such a scenario, the conventional BOTMA can be run by neglecting the site effect, 360 so by imposing that  $\cos(\phi(t)) = 1$ ,  $\forall t$ . The (incorrect) noise-free measurement model is 361 then

 $\cos(\alpha(t)) = \cos(\theta(t) - h(t)).$ 

The results are plotted in Figure 8. Obviously a huge bias appears, leading to an av-364 eraged NEES equal to 1960. More precisely, the bias on the components of the reduced 365 state vector is (-3062.8 -2319.9 14.4 15.8)<sup>T</sup>, rendering the BOTMA inoperative. 366 Clearly, the conventional BOTMA cannot be recommended for the near field. This justifies 367 a posteriori the interest in taking the site effect and the nature of the wave ray into account, 368 as previously pointed out in the introduction of [23]. 369

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Figure 8. The cloud (in green) of the 500 initial positions estimates given by the classic BOTMA374together with the 90%-confidence ellipse.375

## 5. TMA from the direct path and the bottom-reflected path

We assume in this section that the sound wave emitted by the target travels on the 377 direct path and the bottom-reflected path. 378

#### 5.1. Observability

## **Theorem 3.** Let a linear antenna and a source both be in CV motion.

*The antenna acquires the cosines of the conical angles of the direct path and of the bottom-reflected* 381 *path.* 382

- 1. If the target is broadside to the array, then the set of ghost targets is uncountable: it is composed of all the (virtual) targets at broadside to the array. 384
- If the target is endfire to the antenna, the set of ghost targets is composed of virtual sources at endfire to the antenna.
   385
- 3. If the route of the antenna and the route of the target are parallel, then the set of ghost targets 387 is uncountable: at each depth  $z_G$ , there are two ghost targets moving on a cylinder whose axis 388

is the antenna axis, and the radius is a positive scalar  $\beta = \sqrt{\frac{D-z_G}{D-z_T}}$ . The relative ghost target 389

velocity is equal to  $\beta$  times the target's velocity. The initial distance between the ghost image and the center of the antenna is equal to  $\beta$  times the initial distance between the ghost image and the center of the antenna.

4. If the route of the antenna and the route of the target are not parallel, then there are three 393 ghost targets whose motion relative to the antenna is  $P_{OG}(t) = SP_{OT}(t)$ ,  $P_{OG}(t) = \beta P_{OT}(t)$ , 394 and  $P_{OG}(t) = \beta SP_{OT}(t)$ , where **S** is the matrix of the axial symmetry around the line of the 395 antenna, and  $\beta \triangleq \frac{D-z_O}{D-z_T}$ . If the depth of the antenna is equal to the depth of the source, then 396 there is one single ghost target given by  $P_{OG}(t) = SP_{OT}(t)$ . 397

**Proof of Theorem 3.** With no loss of generality, we will again assume that the axis of the 398 sensors array is toward North and that the target is in the half-space where the second 399 component y of any vector is positive. A convenient rotation helps us to be in this case. 400

So the noise-free measurements at time t are 
$$m_1(t) = \frac{y_{OT}(t)}{\sqrt{x_{OT}^2(t) + y_{OT}^2(t) + z_{OT}^2}}$$
, and 401

$$m_2(t) = \frac{y_{0T}(t)}{\sqrt{x_{0T}^2(t) + y_{0T}^2(t) + [2D - (z_T + z_0)]^2}}.$$
402

We have to seek a 5-dimensional state vector  $X_G = (x_G(0) \ y_G(0) \ z_G \ \dot{x}_G \ \dot{y}_G)^T$  defining the trajectory of a ghost target, i.e., producing the same noise-free measurement as X, 404 that is  $m_1(t) = \frac{y_{OG}(t)}{\sqrt{1-2}}$ , and  $m_2(t) = \frac{y_{OG}(t)}{\sqrt{1-2}}$ .

$$\sqrt{x_{OG}^2(t) + y_{OG}^2(t) + z_{OG}^2} \qquad \sqrt{x_{OG}^2(t) + y_{OG}^2(t) + [2D - (z_G + z_O)]^2}$$
  
hence satisfying the two following equalities (in time): 406

$$\frac{y_{OT}(t)}{\sqrt{x_{OT}^2(t) + y_{OT}^2(t) + z_{OT}^2}} = \frac{y_{OG}(t)}{\sqrt{x_{OG}^2(t) + y_{OG}^2(t) + z_{OG}^2}}$$
(4)

$$\frac{y_{0T}(t)}{\sqrt{x_{0T}^2(t) + y_{0T}^2(t) + [2D - (z_T + z_0)]^2}} = \frac{y_{0G}(t)}{\sqrt{x_{0G}^2(t) + y_{0G}^2(t) + [2D - (z_G + z_0)]^2}}$$
(5)

under the constraint that  $z_G$  is in [0, D].

Case 1:  $y_{0T}(t)$  is a zero function, i.e.  $\forall t \ y_{0T}(t) = 0.$  408

The target is broadside to the antenna, so any ghost targets will be too (see Case 1 in the proof of theorem 1). 409

Case 2:  $y_{OT}(t)$  is not a zero function.

From Case 2 of the proof of theorem 1, there is a positive scalar  $\beta$  such that  $y_{OG}(t) = \beta y_{OT}(t)$ . 413

$$(4) \Leftrightarrow \sqrt{x_{OG}^2(t) + y_{OG}^2(t) + z_{OG}^2} = \beta \sqrt{x_{OT}^2(t) + y_{OT}^2(t) + z_{OT}^2} \Leftrightarrow [x_{OG}^2(t) + y_{OG}^2(t) + z_{OG}^2] = \beta^2 [x_{OT}^2(t) + y_{OT}^2(t) + z_{OT}^2]$$
(6)

$$(5) \Leftrightarrow [x_{0G}^2(t) + y_{0G}^2(t) + [2D - (z_G + z_0)]^2] = \beta^2 [x_{0T}^2(t) + y_{0T}^2(t) + [2D - (z_T + z_0)]^2]$$
(7)

Subtracting (7) from (6), we get  $z_{0G}^2 - [2D - (z_G + z_0)]^2 = \beta^2 [z_{0T}^2 - [2D - (z_T + z_0)]^2].$ Now, we simplify the expressions of these two terms: 416  $z_{0G}^2 - [2D - (z_G + z_0)]^2 = -4(D - z_G)(D - z_0)$  $z_{0T}^2 - [2D - (z_T + z_0)]^2 = -4(D - z_T)(D - z_0).$ 

We deduce from this that

$$\beta = \sqrt{\frac{D - z_G}{D - z_T}} \tag{8}$$

Note that  $\beta = 1$  iif  $z_G = z_T$ .

Since  $x_{OG}^2(t) - \beta^2 x_{OT}^2(t)$  is a polynomial function of degree 2, (9) is equivalent to

$$(x_{OG}^2(0) = \beta^2 x_{OT}^2(0) + \beta^2 z_{OT}^2 - z_{OG}^2$$
(10-a)

$$\begin{aligned} x_{OG}(0)\dot{x}_{OG} &= \beta^2 x_{OT}(0)\dot{x}_{OT} \\ \dot{x}^2 &= \beta^2 \dot{x}^2 \end{aligned} \tag{10-b}$$

$$x_{\bar{o}G} = \beta^2 x_{\bar{o}T} \tag{10-c}$$

 First case  $\dot{x}_{0T} = 0$  422

 Eq. (10-c) implies that  $\dot{x}_{0G} = 0$ .
 423

 Consequently, for any  $z_G$  in [0,D], the vector 424

  $X_{0G} = (\pm \sqrt{\beta^2 x_{0T}^2(0) + \beta^2 z_{0T}^2 - z_{0G}^2} \ \beta y_{0T}(0) \ z_{0G} \ 0 \ \beta \dot{y}_{0T})^T$  (with  $\beta = \sqrt{\frac{D-z_G}{D-z_T}}$ ) de- 425

## <u>Second case</u> $\dot{x}_{0T} \neq 0$

Using (10-c), and squaring the terms of (10-b), we get  $x_{OG}^2(0) = \beta^2 x_{OT}^2(0)$ . 428 Reporting this in (10-a), we obtain finally  $\beta^2 z_{OT}^2 = z_{OG}^2$ , i.e. 429

$$\beta^2 = \left(\frac{z_{OG}}{z_{OT}}\right)^2 \tag{11}$$

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If  $z_T = z_0$ , then  $z_G = z_0$ . In this case,  $\beta = 1$ , and consequently  $y_{0G}(t) = y_{0T}(t)$  and 430  $x_{OG}^2(t) = x_{OT}^2(t)$  from (9). The source's trajectory is observable up to the axial symmetry 431 around the (Oy)-axis. 432

Eq. (8) and (11) give us 
$$\frac{D-z_G}{D-z_T} = \left(\frac{z_{OG}}{z_{OT}}\right)^2$$
. 433  
The unknown  $z_G$  is hence a root of the following equation of degree 2: 434

The unknown  $z_G$  is hence a root of the following equation of degree 2:

$$(z_G - z_0)^2 - \frac{(z_T - z_0)^2}{D - z_T} (D - z_G) = 0 \quad \text{which} \quad \text{can} \quad \text{be} \quad \text{expanded} \quad \text{as} \quad \text{follows:} \quad 435$$
$$z_G^2 + z_G \left[ -2z_0 + \frac{(z_T - z_0)^2}{D - z_T} \right] - \frac{D(z_T - z_0)^2}{D - z_T} + z_0^2 = 0. \quad 436$$

Of course,  $z_T$  is a root of this equation. For this value,  $z_G = z_T$ , hence  $\beta = 1$ . 437

The second root 
$$(z_G \text{ itself})$$
 is hence  $2z_O - z_T - \frac{(z_T - z_O)^2}{D - z_T} \triangleq z_G$ . We can check readily that 438

$$z_{G} - z_{O} = z_{O} - z_{T} - \frac{(z_{T} - z_{O})}{D - z_{T}} = \frac{(z_{O} - z_{T})(D - z_{T}) - (z_{T} - z_{O})}{D - z_{T}}.$$
439  
Hence,  $\frac{z_{G} - z_{O}}{D - z_{T}} = \frac{z_{O} - D}{D - z_{T}}$  (which is negative).
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Hence, 
$$\frac{1}{z_T - z_O} - \frac{1}{D - z_T}$$
 (which is negative).

We deduce from this that:

- 1. when the target's depth is larger than the array's depth, there is a ghost whose depth 442 is smaller than the array's depth, and vice-versa. 443
- $\beta$ , which is a positive coefficient, is equal to  $\frac{D-z_0}{D-z_T}$ , or 1. 2. 444

Therefore, we have identified three ghost targets:

the first one is defined by  $X_{OG} = (-x_{OT}(0) \quad y_{OT}(0) \quad z_{OT} \quad -\dot{x}_{OT} \quad \dot{y}_{OT})^T$ , 446 the second is defined by  $X_{OG} = (\beta x_{OT}(0) \ \beta y_{OT}(0) \ -\beta z_{OT} \ \beta \dot{x}_{OT} \ \dot{\beta} \dot{y}_{OT})^T$ , 447 а

nd the third by 
$$X_{OG} = (-\beta x_{OT}(0) \quad \beta y_{OT}(0) \quad -\beta z_{OT} \quad -\beta \dot{x}_{OT} \quad \dot{\beta} \dot{y}_{OT})^T$$
.  $\Box$  448

Remark:

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Most of the time, the depth of a submarine vehicle is under the operational constraint: 450 values of  $z_T$  are in  $[0, z_{Max}]$  and  $z_{Max} \ll D$ . For example,  $z_{Max} = 400$  m, while 451 D = 4000m. 452 The proof of the previous theorem must be adapted to this new constraint. 453

First, we use the fact that the function  $u \mapsto f(u) \triangleq 2z_0 - u - \frac{(u-z_0)^2}{D-u}$  is an involution, i.e. 454 f(f(u)) = u.455

Since 
$$f(0) = 2z_0 - \frac{z_0^2}{D}$$
,  $f\left(2z_0 - \frac{z_0^2}{D}\right) = 0.$  456

Now the question is: what are the values of  $z_0$  for which the following inequality holds: 457  $2z_0 - \frac{z_0^2}{D} \le z_{Max}$ , the greatest value of  $z_0$  guaranteeing that  $2z_0 - \frac{z_0^2}{D-z_T} \le z_{Max}$  is 458  $D - D \sqrt{1 - \frac{z_{Max}}{D}}$  (which is less than  $z_{Max}$ ). 459

If  $z_0 > D - D \sqrt{1 - \frac{z_{Max}}{D}}$ , then  $z_G > z_{Max}$ . In this case, there is a unique ghost target given 460 by  $X_G = (-x_T(0) \ y_T(0) \ z_T \ -\dot{x}_T \ \dot{y}_T)^T$ . 461

If  $z_0 \le D - D \sqrt{1 - \frac{z_{Max}}{D}}$ , then  $z_G \le z_{Max}$ . In this case, there are three ghost targets: 462

one is defined by  $X_G = (-x_T(0) \quad y_T(0) \quad z_T \quad -\dot{x}_T \quad \dot{y}_T)^T$ , the second is defined by  $X_T = (\beta r_T(0) \quad \beta v_T(0) \quad f(z_T) \quad \beta \dot{x}_T \quad \beta \dot{y}_T)^T$ 463

the second is defined by 
$$X_G = (\beta x_T(0) \ \beta y_T(0) \ f(z_T) \ \beta \dot{x}_T \ \beta \dot{y}_T)^T$$
, 464

and the third by  $X_G = (-\beta x_T(0) \ \beta y_T(0) \ f(z_T) \ -\beta \dot{x}_T \ \beta \dot{y}_T)^T$ . 465 466

Note that the operational constraint allows us to benefit from the following range:  $\frac{D-z_{Max}}{D} \le \beta \le \frac{D}{D-z_{Max}}$ . For example, when  $z_{Max} = \frac{D}{10}$ ,  $0.9 \le \beta \le 1.11$ . Consequently, the ghost target is very close to the target of interest.

## 5.2. Estimation of the trajectory

This section is devoted to the estimation of the target's trajectory, or in other words, 470 the estimation of X with  $t^* = 0$  (the first time). Before going into detail, we compute the 471 so-called Cramér-Rao lower bound to evaluate the asymptotical performance of any un-472 biased estimator. 473

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We have considered two typical scenarios. In both, the array is assumed motionless 474 (or, more realistically, all the mobiles are referenced to it) at the depth  $z_0 = 200$  m, and 475 the state vector defining the target's trajectory is given by the state vector 476  $X = (5000 \ 7000 \ 100 \ 2.83 \ 2.83)^T$ . The standard deviation of the measurement is 477  $\sigma = 1.7 \ 10^{-2}$ . The total duration of the scenario is 5 min, and the sampling time is 478  $\Delta t = 4$ s; consequently, the number of measurement couples is N = 75. 479

In the first scenario, the bottom depth is D = 2000 m, while in the second 480D = 4000m. 481

Note that in the first scenario  $\beta = 0.89$  and in the second one  $\beta = 0.97$ . The ghost 482 target is hence very close to the target of interest. 483

#### 5.2.1. Estimability

As pointed in section 1, the state vector X is "estimable" if its asymptotical perfor-485 mance given by the CRLB is compatible with the physical constraints. Typically, if the 486 minimum standard deviation defined by the square root of the third diagonal element of 487 the CRLB (hence of the depth) is much larger than the depth, then X is declared non-488 estimable. 489

1. First scenario

For	this	scenari	o, th	e s	square	root	t of	the	diagonal	of	the	CRLB	491
$\sigma_{CRLB}$	3 = (1.	16 10 <sup>6</sup>	1.59	10 <sup>6</sup>	8.22	10 <sup>5</sup>	637.9	646	$(.1)^T$ .				492

2. Second scenario

> With the bottom depth, things not much better, since 494 are  $\sigma_{CRLB} = (6.59 \ 10^5 \ 8.96 \ 10^5 \ 9.73 \ 10^5 \ 352.6$  $362.2)^{T}$ .

In both cases, the minimum standard deviations are huge. We can conclude that the state 496 vector is not estimable. Computations of minimum standard deviations were made for 497 various scenarios; in all, the state vector is not estimable. 498

A palliative of this is to fix the depth of the source at an arbitrary and realistic value, 499 compute the CRLB the reduced say  $Z_{AS}$ and of state vector 500  $X_r \triangleq (x_T(0) \ y_T(0) \ \dot{x}_T \ \dot{y}_T)^T$  when we assume that  $z_T = z_{AS}$ . For example, for 501  $z_{As} = 300$ m, the minimum standard deviations are 502

 $\sigma_{CRLB} = (281.17 \ 319.37 \ 1.78 \ 2.02)^T$  for the first scenario, and

 $\sigma_{CRLB} = (130.1 \ 115.3 \ 0.80 \ 0.71)^T$  for the second one.

Therefore, we propose to estimate the state vector with this hypothesis ( $z_{As} = 300$ m). 505 In so doing, we introduce a bias. The next subsection gives us the result of the 500 Monte 506 Carlo simulations. 507

## 5.2.2. Monte Carlo simulations

The computation of the maximum likelihood estimator (MLE) is made with the 509 Gauss-Newton routine. No numerical issue was encountered. 510

First scenario 1.

The performance of the MLE is summarized in Table 5. We have numerically com-512 puted the bias and the empirical standard deviation (given respectively in the second and 513 third column of the table). We can see that the empirical standard deviation is very close 514 to the one given by the CRLB. However, as expected, the MLE is biased (of course, there 515 is no bias if we choose  $z_{As} = z_T$ ). In Figure 9, the 90%-confidence ellipse has been drawn, 516 together with the cloud of the 500 estimates (in pink). 517

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X <sub>r</sub>	Bias	$\sigma_{samp}$	$\sigma_{CRLB}$
5000 m	401.12	281.85	281.17
7000 m	557.24	330.87	319.37
2.83 m/s	0.13	1.58	1.78
2.83 m/s	0.12	1.81	2.02

Table 5. Performance of the estimator of the reduced state vector.



**Figure 9**. The location of the sensor array (in black), the cloud of the 500 estimates and the 90%confidence ellipse when D = 2000m,  $z_{As} = 300m$ , and  $z_T = 100m$ . The symmetrical cloud is plotted too. 529

2. Second scenario: Bottom depth D = 4000m.

Again, the performance is presented in Table 6. The bias of the estimator is similar to 531 the one obtained for the first scenario. Only the empirical standard deviations of  $(x_T(0) \ y_T(0))^T$  are larger than the one computed from the CRLB. However, figure 10 533 shows us that the cloud of estimates is close to the true value and not spread. 534

Table 6. Performance of the estimator.

$X_r$	Bias	$\sigma_{samp}$	$\sigma_{CRLB}$
5000 m	306.81	219.28	130.08
7000 m	432.46	276.61	115.26
2.83 m/s	0.18	0.74	0.80
2.83 m/s	0.18	0.66	0.71

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Figure 10. The location of the sensor array (in black), the cloud of the 500 estimates of the initial positions, and the 90%-confidence ellipse when D = 4000m,  $z_{As} = 300m$ , and  $z_{As} = 100m$ , together with the symmetrical cloud.

What is remarkable is the short duration and still the very good performance (in 542 terms of accuracy) of the result. Numerous simulations (not reported here) were per-543 formed; all confirm the correct performance of the MLE. The shortness of the scenario is 544 crucial, because everything that we propose here works properly under the condition that 545 the sea bottom is a plane. During a short scenario, this assumption is likely. 546

## 6. Conclusion

In this paper, conical-angle TMA has been addressed, and various multipaths of 548 sound have been taken into account. The sensor is a line array. Observability was analyzed 549 deeply, allowing all the existing ghost targets to be identified. The main results are that, if 550 the array detects one ray (corresponding to one path), the trajectory is not observable: the 551 set of ghost targets is composed of trajectories that are homothetic to the trajectory of the 552 target of interest, and their symmetrical images by the axial symmetry around the line 553 array. If the array detects two rays (corresponding to two different paths), the number of 554 ghost targets is reduced to three (except when the target is endfire or broadside to the 555 antenna). When the antenna maneuvers, the target's trajectory is observable (apart from 556 the special scenario where there is one single ghost target). Even for "observable" scenar-557 ios, the depth of the target is not estimable (its asymptotical standard deviation is huge). 558 In these cases, we give a non-restrictive palliative that allows us to provide estimates close 559 to the truth. 560

In the future, in this context, many problems remain to be faced: identification of the paths, maneuvering targets, and fusion of data collected by other sensors as in [28]. The 562 problem of seeking a "good" maneuver of the observer, as it was solved in 2D environ-563 ment [29-31] will be addressed in the future. Some of these are already under investiga-564 tion. 565

#### Appendix A

**Proof of Theorem 2**. The proof is made when the first leg is towards North (as previously). Hence  $V_1 = \begin{bmatrix} 0 \\ v \end{bmatrix}$ ,  $V_2 = \begin{bmatrix} v \sin(\alpha) \\ v \cos(\alpha) \end{bmatrix}$ ,  $S_1 = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$ , and  $S_2 = \begin{bmatrix} -\cos(2\alpha) & \sin(2\alpha) \\ \sin(2\alpha) & \cos(2\alpha) \end{bmatrix}$ . Moreo-567 568 ver, we assume  $\alpha \neq k\pi$ . 569 From Theorem 1, we have to consider the four following cases for each leg: 570 the target is broadside to the antenna, 571 . the target is endfire to the antenna, 572 the target has the same heading as the array (but is not endfire to it), 573

the other cases.

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Note that if the target is in case (1) during the first leg, then in case (2) during the second 575 one (provided that this situation is possible), the conclusion about observability will be 576 the same as if the target is in case (2) during the first leg, then in case (1) during the second 577 leg. To be convinced of this, we have just to reverse the time in the equation. This remark 578 allows us to shorten the proof. 579

Case 1: the target is broadside to the antenna during the first leg.

Hence  $P_T(0) = \begin{bmatrix} x_T(0) \\ 0 \end{bmatrix}$ , and  $V_{0T} = \begin{bmatrix} c_T \\ 0 \end{bmatrix}$  during the first leg, which implies  $P_{0T}(t) = \begin{bmatrix} x_T(0) + tc_T \\ 0 \end{bmatrix}$  for  $t \le \tau$ . The ghost targets are also in the broadside, hence 581 582  $P_{OG}(t) = \begin{bmatrix} x_G(0) + tc_G \\ 0 \end{bmatrix} \text{ and } V_{OG} = \begin{bmatrix} c_G \\ 0 \end{bmatrix} \text{for } t \le \tau.$ 583

Can the target be endfire to the antenna? If so, the target has the same heading as the 584antenna during the second leg or, in other words,  $V_T - V_2 = \lambda V_2$ , and  $P_{0T}(t)$ , which is 585 equal to  $P_{0T}(t) = P_{0T}(\tau) + (t - \tau)(V_T - V_2)$ , is collinear with  $V_2$ , whenever  $t \ge \tau$ . The first 586 condition cannot be satisfied since  $V_T = \begin{bmatrix} c_T \\ v \end{bmatrix}$ , and  $V_2 = \begin{bmatrix} \pm v \\ 0 \end{bmatrix}$ . There is no ghost. 587

We skip the case where the target is in case (3) during the second leg. This will be treated 588 later. Therefore, we have now to consider the other cases during the second leg. There are 589 two possibilities for the ghost targets: those whose trajectories are defined by (i) 590  $P_{OG}(t) = \beta P_{OT}(t)$ , and those whose trajectories are given by (ii)  $P_{OG}(t) = \beta S_2 P_{OT}(t)$ , both 591 for  $t \geq \tau$ . 592

The derivative of (i) is 
$$V_G - V_2 = \beta V_T - \beta V_2$$
, hence  $V_G = \beta V_T + (1 - \beta) V_2$ .  

$$\Leftrightarrow \begin{bmatrix} c_G \\ c_G \end{bmatrix} = \beta \begin{bmatrix} c_T \\ c_T \end{bmatrix} + (1 - \beta) v \begin{bmatrix} \sin(\alpha) \\ c_T \end{bmatrix}.$$
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$$\bigoplus_{v} \begin{bmatrix} v \end{bmatrix} = \beta \begin{bmatrix} v \end{bmatrix} + (1 - \beta) v \begin{bmatrix} \cos(\alpha) \end{bmatrix}'$$
  
which implies that  $(1 - \beta) \cos(\alpha) = 1 - \beta$ . Since  $\cos(\alpha) \neq 1$ ,  $\beta = 1$ . Th

ere is no ghost 595 given by (i). 596

The derivative of (ii) is 
$$V_G - V_2 = \beta \mathbf{S}_2 V_T - \beta V_2$$
, hence  $V_G = \beta \mathbf{S}_2 V_T + (1 - \beta) V_2$ .  

$$\begin{bmatrix} c_G \\ c_T \cos(2\alpha) + v \sin(2\alpha) \end{bmatrix} = \begin{bmatrix} c_T \cos(2\alpha) + v \sin(2\alpha) + v \sin(2\alpha) \end{bmatrix} = \begin{bmatrix} c_T \cos(2\alpha) + v \sin(2\alpha) + v \sin(2\alpha) \end{bmatrix} = \begin{bmatrix} c_T \cos(2\alpha) + v \sin(2\alpha) + v \sin(2\alpha) + v \sin(2\alpha) \end{bmatrix} = \begin{bmatrix} c_T \cos(2\alpha) + v \sin(2\alpha) + v \sin(2\alpha) + v \sin(2\alpha) \end{bmatrix} = \begin{bmatrix} c_T \cos(2\alpha) + v \sin(2\alpha) + v \sin(2\alpha) + v \sin(2\alpha) \end{bmatrix} = \begin{bmatrix} c_T \cos(2\alpha) + v \sin(2\alpha) + v \sin(2\alpha) + v \sin(2\alpha) + v \sin(2\alpha) \end{bmatrix} = \begin{bmatrix} c_T \cos(2\alpha) + v \sin(2\alpha) + v \sin(2\alpha) + v \sin(2\alpha) + v \sin(2\alpha) \end{bmatrix} = \begin{bmatrix} c_T \cos(2\alpha) + v \sin(2\alpha) \end{bmatrix} = \begin{bmatrix} c_T \cos(2\alpha) + v \sin(2\alpha) + v \sin(2$$

$$\Leftrightarrow \begin{bmatrix} v \end{bmatrix} = \beta \begin{bmatrix} c_T \sin(2\alpha) + v \cos(2\alpha) \end{bmatrix} + (1 - \beta) v \begin{bmatrix} \cos(\alpha) \end{bmatrix}.$$
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We deduce that  $\beta = v \frac{1}{c_T \sin(2\alpha) + v \cos(2\alpha) - v \cos(\alpha)}$ 

One ghost exists if  $c_T \sin(2\alpha) + v \cos(2\alpha) - v \cos(\alpha)$  is a positive quantity. If so, we then compute  $c_c$ . There is one ghost at most.

Case 2: the target is endfire to the antenna during the first leg. Hence  $P_T(0) = \begin{bmatrix} 0 \\ y_T(0) \end{bmatrix}$ , and  $V_{0T} = \begin{bmatrix} 0 \\ c_T \end{bmatrix}$ , which implies that  $P_{0T}(t) = \begin{bmatrix} 0 \\ y_T(0) + tc_T \end{bmatrix}$  for 603  $t \le \tau$ . During this first leg, the ghost targets are also endfire to the antenna, so  $P_{0G}(t) = \begin{bmatrix} 0 \\ y_G(0) + tc_G \end{bmatrix}$  for  $t \le \tau$ , and  $V_G - V_1 = \begin{bmatrix} 0 \\ c_G \end{bmatrix}$ . 604 605

Again, we skip the case where the target is in case (3) during the second leg. This will be 606 treated later. So, we have now to consider the other cases during the second leg. There are 607 two possibilities for the ghost targets: those whose trajectories are defined by (i) 608  $P_{OG}(t) = \beta P_{OT}(t)$  and those whose trajectories are given by (ii)  $P_{OG}(t) = \beta S_2 P_{OT}(t)$ , both 609 for  $t \geq \tau$ . 610

The derivative of (i) is 
$$V_G - V_2 = \beta V_T - \beta V_2$$
, hence  $V_G = \beta V_T + (1 - \beta) V_2$ . 611

$$\Leftrightarrow [c_G + v] = \beta [c_T + v] + (1 - \beta)v [\cos(\alpha)].$$
We deduce that  $\beta = 1$ . There is no ghost.
612

We deduce that  $\beta = 1$ . There is no ghost. us  $V_G - V_2 = \beta S_2 V_T - \beta V_2$ , Now, differentiating (ii) hence gives 614  $V_G = \beta \boldsymbol{S}_2 V_T + (1 - \beta) V_2.$ 615

$$\Leftrightarrow \begin{bmatrix} 0\\ c + v \end{bmatrix} = \beta(c_T + v) \begin{bmatrix} \sin(2\alpha)\\ \cos(2\alpha) \end{bmatrix} + (1 - \beta)v \begin{bmatrix} \sin(\alpha)\\ \cos(\alpha) \end{bmatrix}.$$
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$$\Rightarrow \beta(c_T + \nu) \sin(2\alpha) + (1 - \beta)\nu \sin(\alpha) = 0.$$
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We deduce that  $\beta = -v \frac{\sin(\alpha)}{c_T \sin(2\alpha) + v \sin(2\alpha) - v \sin(\alpha)}$ . One ghost exists if 618  $c_T \sin(2\alpha) + v \sin(2\alpha) - v \sin(\alpha)$  is a negative quantity. If so, we then compute  $c_G$ . There 619 is one ghost at most. 620

Case 3: the target has the same heading as the array (but is not endfire to it) 621 As in case (2),  $V_{0T} = \begin{bmatrix} 0 \\ c_T \end{bmatrix}$ , but here, the first component of  $P_{0T}(t)$  is not zero. Hence, 622  $V_T = \begin{bmatrix} 0 \\ c_T + v \end{bmatrix}$ , and the target cannot be endfire to the antenna during the second leg. In 623 this case,  $V_{0G} = \begin{bmatrix} 0 \\ \beta c_T \end{bmatrix}$ , hence  $V_G = \begin{bmatrix} 0 \\ \beta c_T + v \end{bmatrix}$ . 624

Can the target be broadside to the antenna? The answer is positive if  $V_2 \perp V_T$  and  $V_{0T}$  is 625 collinear to  $P_{0T}(t)$ , when  $t \ge \tau$ . The first condition implies that  $V_2 = \begin{bmatrix} \pm v \\ 0 \end{bmatrix}$ . Since 626  $P_{0T}(t) = P_{0T}(\tau) + (t - \tau)(V_T - V_2)$ , the second condition is satisfied if  $P_{0T}(\tau)$  is collinear 627 to  $V_T - V_2$ . This is not the case when the first component of  $P_{0T}(\tau)$  is zero, while the first 628 component of  $V_T - V_2$  is  $\pm v$ . 629

So, we have now to consider the other cases during the second leg. There are two possibilities for the ghost targets: those whose trajectories are defined by (i)  $P_{OG}(t) = \beta P_{OT}(t)$  631 and those whose trajectories are given by (ii)  $P_{OG}(t) = \beta S_2 P_{OT}(t)$ , both for  $t \ge \tau$ . 632

The derivative of (i) is  $V_G - V_2 = \beta V_T - \beta V_2$ , hence  $V_G = \beta V_T + (1 - \beta)V_2$  or, in other 633 words, 634

$$\begin{bmatrix} 0\\ \beta c_T + v \end{bmatrix} = \begin{bmatrix} 0\\ \beta (c_T + v) \end{bmatrix} + (1 - \beta) v \begin{bmatrix} \sin(\alpha)\\ \cos(\alpha) \end{bmatrix}.$$
 We conclude that  $\beta = 1$ , i.e. there is no 635 ghost.

If 
$$P_{OG}(t) = \beta S_2 P_{OT}(t)$$
, then  $V_G = \beta S_2 V_T + (1 - \beta) V_2$  637

$$\begin{vmatrix} 0\\ \beta c_T + v \end{vmatrix} = \beta (c_T + v) \begin{vmatrix} \sin(2\alpha)\\ \cos(2\alpha) \end{vmatrix} + (1 - \beta) v \begin{vmatrix} \sin(\alpha)\\ \cos(\alpha) \end{vmatrix}.$$
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This implies that  $\alpha = 0$ , which must be rejected by assumption. There is no ghost. The other cases:

In the other cases, the motion of ghost targets is defined during the first leg by when  $t \leq \tau$ ,

(P

$$P_{OG}(t) = \beta_1 P_{OT}(t) \tag{12-a}$$

or 
$$P_{OG}(t) = \gamma_1 \mathbf{S}_1 P_{OT}(t)$$
 (12-b)

and during the second leg by when  $t \ge \tau$ ,

$$\mathcal{P}_{OG}(t) = \beta_2 \mathcal{P}_{OT}(t) \tag{13-a}$$

(or 
$$P_{OG}(t) = \gamma_2 \mathbf{S}_2 P_{OT}(t)$$
 (13-b)

Hence at time  $\tau$ , the position of a ghost target is

$$P_{OG}(\tau) = \beta_1 P_{OT}(\tau) \tag{14-a}$$

$$\text{or } P_{OG}(t) = \gamma_1 \mathbf{s}_1 P_{OT}(t) \tag{14-b}$$

and 
$$P_{OG}(\tau) = \beta_2 P_{OT}(\tau) \tag{15-a}$$

or 
$$P_{0G}(\tau) = \gamma_2 S_2 P_{0T}(\tau)$$
 (15-b)

Of course, (14-a) and (14-b) are not compatible, and neither are (15-a) and (15-b).646Now, let us show that (14-a) is not compatible with (15-b):647Indeed, if  $P_{OG}(\tau) = \beta_1 P_{OT}(\tau) = \gamma_2 S_2 P_{OT}(\tau)$ , then648

$$\frac{\beta_1}{\gamma_2} P_{0T}(\tau) = \mathbf{S}_2 P_{0T}(\tau)$$
 (16)

Eq. (16) implies that  $P_{0T}(\tau)$  is an eigenvector of  $S_2$ , with the eigenvalue  $\frac{\beta_1}{\gamma_2}$ . Since  $\frac{\beta_1}{\gamma_2}$  is 649 positive, this eigenvalue is equal to 1, i.e.  $\gamma_2 = \beta_1$ . Hence  $P_{0T}(\tau)$  is in the second leg. 650 Hence, the set of ghost targets is reduced to those whose positions at time  $\tau$  are given by (14-a) or (14-b), and (15-a). Now suppose that a ghost target satisfies (14-b) and (15-a). By 652

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the same computation, we conclude that  $P_{OT}(\tau)$  is in the first leg, which is impossible 653 since  $P_{OT}(\tau)$  is in the second leg. 654 We have proven that (14-a) and (15-a) only are compatible. It follows that a ghost target 655 verifies these two equalities (given by (12-a) and (13-a)): 656

 $P_{OG}(\tau) = \beta_1 P_{OT}(\tau) = \beta_2 P_{OT}(\tau).$ 

Hence,  $\beta_1 = \beta_2$ .

Now taking the derivative of the two members of (12-a) and of (13-a), we obtain  $V_G = \beta_1(V_T - V_1) + V_1 = \beta_1(V_T - V_2) + V_2$ , which is equivalent to  $(\beta_1 - 1)(V_2 - V_1) = 0$ .

Since 
$$V_2 \neq V_1$$
,  $\beta_1 = 1$ 

Putting this value in (12-a) or in (13-a), we finally get  $P_G(t) = P_T(t)$ . The "ghost" is the 663 target of interest. In conclusion, there is no ghost target.  $\Box$  664

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